Perspectives on possibility modals[1]

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Abstract. Possibility modals are standardly treated as expressing compatibility with contextually relevant information, rules, goals, desires, relevant circumstances, or combinations thereof; equivalently, existential quantification over relevantly accessible worlds. Problems are well-known especially for dynamic and teleological modals, but solutions are typically tailored to one specific modal flavor. I propose a unified semantics in terms of plural existential quantification that derives stronger readings when actions under the control of a relevant agent are salient, but trivializes to the familiar compatibility semantics otherwise.

Keywords: modality, possibility, deontic, priority, teleological, goals.

1. Introduction

Natural language modality is broadly understood as a cover terms for expressions that 'concern[s] situations which need not be real' (Portner, 2009) or serve to 'abstract away from the here and now' (Kaufmann and Kaufmann, 2015). The category is widely taken to be represented exemplarily by auxiliaries like English *may* and *must*. Following the tradition of classical modal logic, they are standardly interpreted as expressing compatibility and entailment, respectively, with respect to information, rules, goals, desires, relevant circumstances, or combinations thereof. The more recent decades have seen many refinements of this picture, including for instance a growing interest in distinctions in strength between *must* and *should* (which, intuitively, both express entailment) as well as differences regarding the speech acts performed (performative vs. descriptive modality, and relevant differences between *must* and *have to*). Possibility modals have received a significant amount of attention in the epistemic domain (see von Fintel and Gillies, 2011) and in connection with free choice inferences. While problems with the compatibility semantics are well-known, discussions usually focus on specific flavors (apart from epistemic modality, mostly ability modals and certain instances of teleological modality), and alternative proposals are usually tailor-made for specifc modal flavors. Overall, the compatibility semantics remains the standard solution (with the possible exception of ability modals). In this paper, I would like to propose a unified semantics for possibility modals across priority and dynamic flavors. I suggest to treat them as plural existential quantifiers whose domains are restricted through an additional contextually determined parameter, which trivializes to the regular compatibility semantics unless the modality in question is concerned with action choices for a relevant agent. I begin with a short introduction of the received framework in Sect. 2, continue with specific problems for teleological, dynamic, and deontic modals in Sect. 3 and an idea for a unified solution in Sect. 4, with an exploration of the implications and a possible implemenation in a causal network in Sect. 4.3. I evaluate predictions and findings in connection with negation in Sect. 5 and conclude in Sect. 6.

[1]For discussion of data and theory, I thank the audiences of the workshops *Future Developments in Formal Semantics*, Sapporo University, August 18-19, 2025, *Modality and Scalarity*, UConn, September 15-17, 2025, *Sinn und Bedeutung 30*, as well as the participants of *LING 5410 Semantics I*, UConn, Fall 2025.

2. The received view

The standard analysis of natural language modality in linguistic semantics builds on the Kripkean interpretation of possibility and necessity operators with respect to a model M that consists of a set of possible worlds W, an accessibility relation $R \subseteq W \times W$, and a valuation that at each world assigns all propositional letters truth-values and preserves the standard interpretations for propositional connectives. Necessity $\Box$ ('must') and possibility $\Diamond$ ('can') are interpreted as universal and existential quantifiers over the possible worlds accessible according to the accessibility relation R.

(1) a. $[\![\Box\phi]\!]^{M,w} = 1$ iff $[\![\phi]\!]^{M,w'} = 1$ for all w' s.t. wRw'.
b. $[\![\Diamond\phi]\!]^{M,w} = 1$ iff $[\![\phi]\!]^{M,w'} = 1$ for some w' s.t. wRw'.

As it stands, different models, each with their own accessibility relation, are needed to interpret modals of different flavors: R can reflect different kinds of epistemic, deontic, or bouletic accessibilty, and more.

For interpreting natural language, the models need to be enriched to allow for different flavors to co-occur and interact. Kratzer (1977; 1978; 1991; 2012, a.o.) emphasizes moreover that modal auxiliaries like *may, can* and *must* do duty across multiple modal flavors. Depending on lexical, temporal, and aspectual properties of the prejacent, as well as the linguistic and extralinguistic context, they can receive epistemic, priority (deontic, teleological, bouletic), or dynamic reading. Negated *can* proves to be one of the most flexible items, allowing the full range of modal flavors:

(2) Mary cannot be at the office (given what we know). epistemic

(3) Priority modality
a. Mary cannot work from home tomorrow (in view of company rules). deontic
b. Mary cannot take the train (in view of her goals). teleological
c. Mary cannot come to the office (in view of my desires). bouletic

(4) Mary cannot sing this song (in view of her internal set-up). dynamic ('ability')

To allow for multiple modals to be interpreted in the same model and honor the use of modal verbs across different modal flavors, Angelika Kratzer lets modal verbs depend on contextually salient conversational backgrounds, functions that assign to each world a set of propositions describing a.o. what is known, required, or desired in the relevant way. Using an interplay of a necessarily consistent conversational background (the modal base) and a second one (the ordering source), which represents a possibly non-realistic ideal, she avoids trivialities that besiege the original system (Kratzer, 1991; Portner, 2009). Kratzer's formulations in terms of human necessity and human possibility are designed to capture also infinite approximation to the ideal (as represented by the ordering source). Where infinite approximation is not a concern, modal base and ordering source can, equivalently, be used to jointly determine an accessibility relation (Kratzer, 1991), reducing the standard framework to a version of the modal logic treatment. Crucially, by these assumptions, possibility modals come out as expressing compatibility between the prejacent and the set of relevantly accessible worlds, or equivalently, existential quantifiers over the set of relevantly accessible worlds (see Sect. 4 for more).

Not all English modals display the full flexibility in terms of modal flavors, and even *can* does not in the absence of negation (epistemic readings are strained or unavailable, in contrast to (2)). Crosslinguistically, modal expressions can be even more restricted in modal flavor (e.g., Nauze, 2008; Narrog, 2009; Rullmann et al., 2008; Kaufmann and Tamura, 2020). However, some modal expressions display (almost) full flexibility, and even those that do not, do not always cut the cake along the same lines. Moreover, they do not necessarily cut it across the lines resulting from modifications proposed to address challenges arising with individual modal flavors. I will therefore uphold the Kratzerian desideratum of a basically unfied semantics across modal flavors, and a forteriori across different types of priority flavors as will become significant in the following.

Finally, it is worth pointing out that modal (semi-)auxliaries are not always distinguished from other expressions with similar meaning, for instance impersonal constructions (*it is possible that,…*) or nominal constructions (*has permission to, has the right to,…*). Various authors have observed interesting differences between these and modal verbs (a.o., Kratzer, 1981; Lauer, 2015; Moltmann, 2020). In the following, I focus on modal (semi-)auxiliaries unless noted otherwise.

By the standard semantics laid out above, possibility modals express that the prejacent is compatible with the relevantly best set of worlds. In the following, I argue that this appears to miss important aspects of their meaning or even gives rise to problematic truth-conditions (as is well-known for ability modals and debated for teleological modals).[2] Before doing so, I would like to mention actuality entailments as one challenge from possibility modals that I take to be orthogonal to the concerns I aim to address.

In many languages, the perfective past tense forms of dynamic and priority modals tend to convey that the eventuality realized by the prejacent was realized; a comparable entailment is absent if the modal carries imperfective aspect (a.o., Bhatt, 1999; Hacquard, 2006; Homer, 2021; Nadathur, 2023, 2025), cf. (5).

(5) Marja {a. pouvait /b. a pu } traverser le lac Nokomis à la nage,
M. {a. can.PST.IMPF /b. has can.PP } cross.INF the lake Nokomis at the swim,
'Mary can.PAST swim across Lake Nokomis.
{a: ok, b: #} …but she never did.' from Nadathur (2023)

While the existing accounts vary in the details, there is general consensus that the phenomenon results from the interplay between possibility modals, aspectual classes, and verbal aspect. In the following, I will set actuality entailments aside and focus on other issues with the standard semantics. It merits mentioning, however, that the tools employed to handle actuality entailments and the challenges I am concerned with overlap partly (see especially Sect. 4.3).

The compability semantics outlined above has the advantage of naturally covering the duality of possibility and necessity modals: Possbility is equivalent to the negation of the necessity of the prejacent's negation,'it is possible that ϕ' is equivalent to 'it is not necessary that not ϕ'. Moreover, for (almost) all modal flavors it is possible to find scenarios in which compability reflects native speaker intuitions. We can understand (6) as informing us that it is compatible

[2] I am setting aside all issues with epistemic modals and will only briefly mention them again in Sect. 6.

with what is required of Mary (e.g., by the course rules) that she leave the door open, and imagine (7) as used by a station attendant to reassure a passenger that buying food is compatible with their goal of reaching a specific train:

(6) Mary can leave the door open.
≈'There is a world compatible with all course rules in which she leaves the door open.'
($\Leftrightarrow_{\text{Duality}}$ 'It is not the case that she has to close the door.')

(7) A: My train leaves at 5pm. Do you think I have time to buy something to eat?
B: You can go and buy something; just make sure you're back by 4:55.
($\Leftrightarrow_{\text{Duality}}$ '[In light of your gal of catching the train] it is not the case that it is necessary that you do not go and buy something.')

Bouletic possibility modals have received less discussion; a concessive case like (8) might qualify. Here, too, compatibility with my desires appears to capture the salient reading successfully.

(8) (For all I care,) Britain can remain a monarchy. (It doesn't matter to me.)

While compatibility with rules, goals, and preferences appears to capture the cases in (6)–(8) adequately, possibility modals are often felt to convey stronger notions as we will see in the following.

3. Compability is not enough

3.1. Dynamic possibility beyond compatibility: Abilities and opportunities

Dynamic possibility modals encompass at least two different notions (Portner, 2009): On the one hand, abilities of agents, learned or innate, and depending on the agents' internal set-up; on the other hand, opportunities, which result from an agent's internal set-up together with facts about specific situations (this is also referred to as circumstantial possibility). Opportunities often seem well-captured in terms of compatibility, as evidenced by (9), an example from Cross (1986):

(9) He can still meet her for dinner tonight.

Abilities seem invariably stronger, compare (10):

(10) She can run a four minute mile. Cross (1986)

On the ability reading, (10) suggests that running a mile does not generally take her more than four minutes, that she has a strategy that allows her to bring it about that she runs a mile in four minutes, or something along these lines. This goes beyond telling us that it is compatible with the circumstances that she runs a four minute mile, which is already true in a world where she normally takes longer but could randomly get lucky and finish faster. This finding has been discussed extensively in the literature (a.o., Kenny, 1975; Brown, 1988; Thomason, 2003; Maier, 2018; Nadathur, 2023). Effects similar to the generic ability ascription result for dynamic possibility modals that seem anchored more tightly to a specific situation. Imagine a situation

in which Mary, a very shaky singer, wants to sing 'First Love'. Similarly to the explicit ability ascription in (11c), the possibility modal in (11a) gives rise to an intuitively false statement; this contrasts with the impersonal adjectival construction in (11b), which is felt to be true.

(11) a. Mary can sing 'First Love' recognizably.
b. It is possible that Mary will sing 'First Love' recognizably.
c. Mary has the ability to sing 'First Love' recognizably.

If dynamic *can* can express compatibility, the contrast between (11a) and (11b) is unexpected.[3] These data suggest that ability *can* is interpreted similarly to *has the ability to*, and conveys more than just compatibility with the relevant circumstances.

Upon closer inspection, opportunities, which in some cases seem well-captured in terms of compatibility with respect to one or more agents' internal setup and relevant circumstances (see (9)), can also convey a stronger meaning. To show this, I want to draw on the situation of the Miners Puzzle, devised originally to highlight information sensitivity of a type of priority modality appearing in practical deliberations and addressing what the salient agents *ought to do*.[4] Here, we are concerned not with the *ought* statements of the original puzzle, but with *can* statements that serve to characterize the actions of interest. The relevant situation is decribed in (12), with the available actions and their consequences depending on the location of the miners summarized in table form:

(12) Ten miners are trapped either in shaft A or in shaft B, but we do not know which. Flood waters threaten to flood the shafts. We have enough sandbags to block one shaft, but not both. If we block one shaft, all the water will go into the other shaft, killing any miners inside it. If we block neither shaft, both shafts will fill halfway with water, and just one miner, the lowest in the shaft, will be killed.

Action	If miners in A	If miners in B
Block shaft A	All saved	All drowned
Block shaft B	All drowned	All saved
Block neither shaft	One drowned	One drowned

In this scenario, the possibility modals in (13) are naturally read as expressing opportunities:

[3]Daniel Hole (p.c.) points out that the contrast could result from an epistemic reading for (11b), which, for a modal auxiliary embedding an eventive predicate, is expected to be unavailable for orthogonal reasons. However, the contrast survives in cases where standard assumptions of epistemic logic would result in infelicitous or marked readings for all conceivable instantiations of the relevant epistemic accessibility:

(i) a. I don't know whether it is possible that Mary will sing 'First Love' recognizably.
b. I don't know whether Mary can sing 'First Love' recognizably.

A reading where *possible* is anchored to the speaker's beliefs should result infelicitous in (ia); anchoring to a salient other epistemic background (according to what the audience believes, according to what her test results say,...) seems unavailable. This suggests that, like *can*, *is possible* receives a dynamic interpretation. Yet, they differ in that I wonder about what's compatible with the current situation (including Mary's abiliites) in (ia), and about Mary's capabilities in the given situation in (ib).

[4]The Miners Puzzle is discussed by Kolodny and MacFarlane (2010), who credit an unpublished manuscript by Derek Parfit, which credits Donald Regan. I am indebted to Stefan Kaufmann (p.c.) for alerting me to the relevance of the Miners Puzzle for this project, especially the interaction with negation in Sect. 5.

(13) a. We can block shaft A, and we can block shaft B.
b. We can block the shaft they are in.

Intuitively, (13a) is true (as it describes the two actions available to us, the relevant agents). In contrast, (13b) seems false on the salient de dicto interpretation of the noun phrase: as we do not know where the miners are, we have no way of ensuring that we block the shaft they are in. This intuition, however, is not captured by the semantics in terms of compatibility. In all worlds that agree with the actual world on the relevant circumstances (and our abilities), the miners are in one of the two shafts, and as we have a choice to block A or B which is independent of the actual location of the miners, there are accessible such worlds where we block the one they are in. It is therefore compatible with the relevant circumstances and our internal make-up that we block the shaft the miners are in ('it is possible that we happen to block the shaft they are in'). The fact that (13b) is perceived to be false suggests that it has a highly salient reading that cannot be captured by the standard semantics in terms of compatibility.

3.2. Teleological modality: Compatbilities and means

Above, we saw that teleological possibility modals can express compatibility ((14) repeats (7)):

(14) A: My train leaves at 5pm. Do you think I have time to buy something to eat?
B: You can go and buy something; just make sure you're back by 4:55.

Other cases, however, appear to convey a stronger connection between the prejacent and the agent's goal(s). Rationale clauses tend to convey that the prejacent describes a means to the goal, resulting in markedness of (15b) (in contrast to (15a)).[5]

(15) To get on the 5pm train, {a. you can bribe the conductor./b. #you can still buy lunch.}

Moreover, only the means reading survives in response to practical questions like (16A), in response to which an answer like (16B) but not (16B') is felicitous.

(16) A: How do I get to Harlem?
B: You can take the A-train.
B': #You can still buy lunch.

Note, however, that the means reading still suggests a form of existential quantification. Unlike what we would find with modals like *must* or *have to*, means providing *can* remains neutral about the existences of alternatives that might also allow the agent to achieve the goal, as shown by the felicity of the follow-ups in (17):

(17) You can take the A-train. {But the B-train is also an option./But I don't know if there are other options.}

[5] I am indebted to Will Rimer (p.c.) for pointing out that the compatibility reading remains marginally available and is easier to access when a limitation is made explicit as in (i):
(i) To get on the 5pm train, you can still buy lunch as long as you're back by 4:55.

Moreover, the prejacent of the means reading does not have to be causally sufficient for the agent's goal (for instance, for (17), at least getting off at the right station is required in addition), it is sufficient that it characterizes a crucial part of a strategy to achieve the goal.

3.3. Deontic: Weak vs. strong permission

As expected from the standard semantics, deontic possibility modals as in (18a) can express compatibility with what is required by the relevant laws or rules. They can, however, also convey that something is a right, as made explicit in (18b). This distinction is discussed as weak permission vs. strong permission (a.o., Hilpinen and McNamara, 2013; Moltmann, 2020).

(18) a. You can stay until 11pm.
b. You have the right to stay until 11pm.

Evidence for the distinction can be found in the restrictions on subsequent commands and permissions: Compatiblity can be delimited by any authority, but rights cannot be delimited by authorities lower than the one who granted the right (Hilpinen and McNamara, 2013). Consider the following exchange, assuming that Older Brother (OB) has authority over Younger Brother (YB), while Mom has authority over both of them:

(19) a. YB reasons: 'Mom only said I need to be there for dinner at 6. So, I can go out and play soccer.' weak permission
b. OB: 'You have to do your homework now.'
c. YB reasons: Ok, so I cannot go out and play soccer.
d. Mom: 'Actually, you can play soccer before you do your homework.'
e. YB reasons: 'I {can/ have the right to} play soccer first.' strong permission
f. OB: #'You have to do your homework before you play.'

Younger Brother's reasoning in (19a) (and (19c)) rests on compabitility with what he is required to do (weak permission) as predicted by the standard semantics. Mom's utterance in (19d) serves to give permission (overwriting Older Brother's command and constituting a challenge for the truth-conditional standard semantics). Younger Brother's reasoning in (19e) reflects not just compatibility, but a right (strong permission). As shown in (19f), Older Brother cannot easily overwrite this.

Strong permission could be considered a failure of compatibility similar to what we observed for dynamic and teleological modals above. However, unlike in the other cases, strong and weak permission both present the prejacent as a course of events the agent could decide to follow; what seems different is how the relevant modal state of affairs has come about. I will tenatively assume that, semantically, deontic *can*-statements express compatibility on both weak and strong permission readings. They differ, however, in the origins of this compatibility: Weak permission reports compatibility resulting from the absence of commands to the contrary. Strong permission either serves to grant a right as an utterance by a relevant authority (performative modality, Kamp 1973; for an overview, Kaufmann, to appear) or reports compatibility that was brought about (or protected) in this way (descriptive modality). In contrast to a modal like *can*, *have the right* describes not only that an eventuality is compatible with the relevant rules or laws, but also that this compatibility is guaranteed by a specific authority. If this is on the right track, the

challenge is to analyze the use of deontic modals to change what is permissible or to provide a guarantee of compatibility, where both of these constitute performative uses, going beyond a mere description of what is permissible (as in descriptive uses). I leave it for future research to determine if the possibility semantics proposed in this paper, beyond being able to still capture compatbility readings, offers tools for addressing challenges with performative modality.

4. The proposal: choosable courses of events

In the previous section, we have seen that dynamic and teleological modals show clear evidence of readings that cannot be captured by the compatibility semantics. These share a crucial role for an agent's control over the relevant course of events: For dynamic modals, we find them as reporting abilities (reflecting a general actionable strategy for realizing something) and as opportunities that come with a crucial component of choosable actions for the agent in that specific situation; for teleological modals, we observe a reading where possibility modals characterize means as specific ways in which the agent can achieve their goal. Deontic modals reflect a difference between compatibility with requirements and rights granted by an authority, which I tentatively set aside as resulting from independent mechanisms of how utterances can modify the sphere of what is permissible. In light of the findings from dynamic and teleological modals, it seems that we need to develop a semantics for possibility modals that is sensitive to whether or not an agent's control over the course of events is in focus. For constellations that are (understood as) beyond the control of the agent, the semantics should reduce to compatibility.

4.1. Possibility modals as plural existentials

I propose to treat possibility modals of at least dynamic and priority flavors as contextually restricted plural quantifiers, cf. (20).

(20) $[\![\textsc{Can}\ \phi]\!]^{c,w} = 1$ iff $\exists W' \in A_c[W' \neq \emptyset \wedge W' \subseteq [\![\phi]\!]^c]$

To make this semantics operational for the cases observed, we will need to address the following two questions: (i) what determines the set of propositions A, and (ii) what is the nature of the embedded ('local') necessity encoded in '$W \subseteq [\![\phi]\!]^c$'.

A plural semantics for existential modals been proposed independently by Klinedinst (2007), who argues that this move allows to unify the free choice reading for disjunctions under possibility modals, cf. (21) (a.o., von Wright, 1968; Kamp, 1973) with distribution of disjunctions under plural indefinites in the individual (shown in (22)) or temporal domain.

(21) You may have beer or wine.
≈ 'You may have beer, and you may have wine.'

(22) [The air in the train was extremely stuffy…] Some passengers became nauseous or had trouble breathing.
= 'Some passengers became nauseous, and some passengers had trouble breathing.'

No distribution effect results under singular indefinites as in (23).

(23) Some passenger became nauseous or had trouble breathing.
≠ 'Some passenger became nauseous and some passenger had trouble breathing.'

The details of how the free choice effect is derived are orthogonal to our interests here,[6] what is crucial is that, in interaction with disjunction, possibility modals pattern with plural indefinites and not with singular indefinites. I will focus first on the individuation of the modal's quantificational domain (set A) in Sect. 4.2, and explore the notion of local necessity in Sect. 5.

4.2. A third parameter

The semantics in (24) (repeating (20)) commits us not only to treating possibility modals as plural quantifiers, but assumes also a restriction on the domain of pluralities it can quantify over (here, a set of sets of propositions A).

(24) $[\![\textsc{Can}\ \phi]\!]^{c,w} = 1$ iff $\exists W' \in A[W' \neq \emptyset \wedge W' \subseteq [\![\phi]\!]^c]$

In our exploration of dynamic and teleological modals, the standard compatibility semantics proved too weak whenever a statement was sensitive to factors under the control of the relevant agent. I therefore propose to instantiate the set of propositions A with a partition on the accessible worlds according to choices the agent can take, i.e. classes of courses of events between which they can choose. So, A is roughly the set of sets of worlds that agree on a particular course of events chosen.[7] The worlds in any cell of A can differ in outcomes, specific ways of implementing a choice, etc. To capture the intuition that, in many cases, the standard compatibility semantics appears satisfactory, I propose that in these cases A is set to the trivial partition given by the identity relation on possible worlds: Quantification over the singleton propositions in the set it corresponds to collapses into quantification over individual worlds. As this is supposed to represent cases that do not intuitively rest on choices taken by a salient agent, I propose to think of these cases as involving as the relevant agent 'nature', who controls everything (giving us the singleton cells of the maximally fine-grained identity partition).

To implement the idea, we will stick closely to Kratzer's doubly relative semantics, which relies on two contextual parameters that represent (i) a stock of relevant facts or an agent's knowledge (the modal base), and (ii) an ideal consisting in preferences, laws, normalcy assumptions, etc. (the ordering source). Both parameters are modeled as conversational backgrounds, functions from worlds to sets of proposition, with metalanguage f commonly used for the modal base, and g for the ordering source. At each world w, the set of propositions assigned by the ordering source $g(w)$ is used to induce a pre-order as defined in (25):

(25) $w' \leq_{g(w)} w''$ iff$_{def}$ $\{p \in g(w) \mid w'' \in p\} \subseteq \{p \in g(w) \mid w' \in p\}$

[6] Klinedinst (2007) proposes local exhaustification over the distributive operator that predicates the remainder of the sentence of each of the elements in the plurality; alternatively, we could treat disjunction as requiring verification on non-empty subsets of a plurality in the sense of Aloni's (2022) Neglect Zero.

[7] Sets like this appear as decision problems in Cariani et al. (2013). This section and the following constitute an attempt to integrate this component of their account for deliberative modals into a unified Kratzer-style semantics and relate it to recent work on dynamic modality that relies on causal networks. I leave it to future work to further assimilate the definitions employed.

Assuming that, for each world compatible with the modal base, there is a unique set of best worlds according to the ordering source (Lewis's 1973 Limit Assumption, see also Kaufmann, 2017) allows us to identify the quantificational domain of a modal given a modal base f and an ordering source g (as determined by the context of utterance c) at a world w as $O(f, g, w)$, cf. (26a). From this, we can define the accessibility relation R_c relevant in the given context (Kratzer, 1991), cf. (26b). We can then treat possibility and necessity modals as the standard operators $\Diamond$ and $\Box$ familiar from modal logic.

(26) a. $O(f, g, w) := \{w' \in \bigcap f(w) \mid \neg\exists w'' \in \bigcap f(w)[w'' <_{g(w)} w']\}$
b. $R_c := \{\langle w, w'\rangle \mid w' \in O(f_c, g_c, w)\}$

To derive the set of sets of propositions that constitutes the quantificational domain for our plural modals, I introduce a third conversational background γ, which assigns to each world w the set of propositions p, whose truth or falsity is under the control of the contextually relevant agent α at w.[8] Specifically, I will assume that α is either the individual whose abilities, choices, or goals matter (often the subject of a priority modal sentence), or else 'nature', in which case the semantics will trivivalize to compatibility.[9]

We can now use $\gamma(w)$ (the set of propositions under the control of the agent at w) to determine classes of worlds that agree exactly on the actions chosen by the relevant agent:

(27) $[w']_{\gamma(w)} := \{w'' \in W \mid \forall p \in \gamma(w)[w' \in p \leftrightarrow w'' \in p]\}$

Whenever we are not concerned with action choices and want possibility modals to express compatibility, we will assume that the role of the additional new parameter is played by the constant function γ_N, reflecting the action choices of nature and assigning to each world w the powerset of W ('nature controls everything'; for all $w \in W$, $\gamma_N(w) = \mathcal{P}(W)$).

As in the original Kratzer framework, quantification will be restricted to worlds that verify the relevant facts (or the relevant stock of knowledge) as assigned to the world of evaluation w by the modal base. For teleological modality specifically, we add to this the proposition that the designated goal is achieved (Huitink, 2005; von Fintel and Iatridou, 2006; Werner, 2006); no considerations of normalcy or convenience should allow us to select worlds where this core goal is not reached.

The worlds compatible with the propositions assigned by the modal base (together with the designated goal where it applies) should be ranked by the ordering source (to retain the achievements of Kratzer's doubly relative modality), but they should also be clustered according to the actions chosen by the relevant agent. The question is thus how clustering by γ and ordering by g should interact to produce the correct readings: Should we group into cells of chosen courses of events first, and find the best ones within each of these (option $\gamma-$first) or should we find the highest ranked first and group them according to actions chosen afterwards (option $g-$-first). The two options for the identification of a modal's quantificational domain $O^3(f, \gamma, w)$ from

[8]Note that this glosses over important questions relating to future orientation and (metaphysical) indeterminacy, see Cariani (2021).

[9]Throughout, I treat modals as propositional operators (syntactically, raising predicates). It remains to be seen if this can be upheld for all dynamic and priority flavors; for discussion, Grano, to appear; Kaufmann, to appear.

modal base f, ordering source g, and agentivity parameter γ, are given in (28) and (29), respectively; for readability, γ–first uses $C(f,\gamma,w)$ as the restriction of the cells of action alternatives by the modal base. (30) repeats (26a) from above.

(28) Option γ-first:
a. $C(f,\gamma,w) := \{\bigcap(f(w) \cup [w']_{\gamma(w)}) \mid w' \in \bigcap f(w)\}$
b. $O^3(f,\gamma,g,w) := \{\{w' \in p[\neg\exists w''[w'' \in p \wedge w'' <_{g(w)} w']\} \mid p \in C(f,\gamma,w)\}$

(29) Option g–first: $O^3(f,\gamma,g,w) := \{O(f,g,w) \cap [w']_{\gamma(w)} \mid w' \in W\}$

(30) $O(f,g,w) := \{w' \in \bigcap f(w) \mid \neg\exists w'' \in \bigcap f(w)[w'' <_{g(w)} w']\}$

The two options differ in whether the ordering source can eliminate cells that represent action alternatives. Intuitions are pretty clear-cut when considering an ordering source that reflects assumptions of normalcy or stereotypicality: a choosable action remains an alternative even if the speaker assumes that that particular action is unlikely to be chosen. This suggests that the clustering into cells of choosable courses of events has to happen first as in (28), so that more or less likely courses of events agreeing on actions chosen are ranked only against each other, and not against alternative action choices (considered) more likely for α to take. At the same time, when γ is set to the trivial γ_N, (28) creates a problem: as the ordering induced by g is considered only on singleton cells, it has no effect.[10] I tentatively assume that (29) is correct, but that by the very nature of reasoning with a parameter representing actions choosable for the agent, the propositions under the control of α should not be subject to normalcy considerations. For this, we have to require that for any $\gamma \neq \gamma_N$, if $[w']_{\gamma(w)}$ has a non-empty intersection with $\bigcap f(w)$, it also has a non-empty intersection with $O(f,g,w)$. While seemingly stipulative in the extended standard framework presented in this section, the separation between normal courses of events and choosable actions receives a more principled analysis if the values of the individual conversational backgrounds are conceptualized as different aspects of a causal network (see Sect. 4.3).

We can now spell out the plural existential semantics as in (31), and I will take $O^3(f,\gamma,g,w)$ to be defined as in (29), with the noted caveat about the contents of g and γ.

(31) $[\![\textsc{Can}\ \phi]\!]^w = 1$ iff $\exists W' \in O^3(f,\gamma,g,w)[W' \neq \emptyset \wedge W' \subseteq [\![\phi]\!]^c]$

Note that the set of propositions $O^3(f,\gamma,g,w)$ correspond to Cariani et al.'s (2013) decision problem. There, the resulting cells are used to model *ought*-statements in practical liberations where actions should only be evaluated according to outcomes shared by entire cells (reflecting what this particular choice would amount to across favorable or less favorable facts the agent can neither know nor control).[11] In this paper, I am only concerned with establishing the quantificational domain of the possibility modals that in these cases describe the options available. The ranking of cells defined by Cariani et al. (2013) could operate on $O^3(f,\gamma,g,w)$ as defined here to interpret *ought*s of practical deliberation.

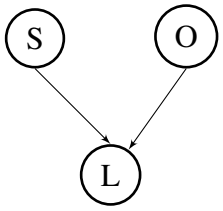
[10] At SuB 30, I adopted a solution similar to (28) and flagged the trivialization of g for γ_N as an open problem.

[11] Technically, Cariani et al. (2013) consider the set of choosable actions itself a conversational background δ; its restriction by the modal base yields $[f \mid \delta]$ applied to w corresponds to $C(f,\gamma,w)$ as defined here. They abstract away from other ordering sources that, like considerations of normalcy, should rank individual worlds rather than entire cells.

4.3. Implementing choosable action

So far, I have treated γ as a blackbox that to each world assigns the set of propositions whose truth or falsity is under the control of an agent. In line with recent works that draw on causal networks for the modeling of natural language semantics, especially in the domain of priority and dynamic modality (e.g., Nadathur, 2023; Santorio, ta), I would like to sketch a way of relating the proposal from the previous section to the results in this literature.

I assume a basic causal model, where causal dependencies can be represented as an acylic graph with structural equations to capture the nature of the dependencies (a.o., Pearl, 2000; Kaufmann, 2005; Schulz, 2007). Nodes (variables) correspond to the set of causally relevant bipartitions $X = \{x, \neg x\}$, and are divided into exogenous (given) variables and endogenous variables (value derived by the causal model). Edges represent immediate causal ancestry. Consider the famous toy example from Goodman (1947), involving a match that will or will not strike depending on being struck in the presence or absence of oxygen. The nature of causal dependence assumed is represented in the table to the right of the graph that specifies the truth-values for the endogenous variables depending on their causal ancestors:



s…The match is struck.
o…Oxygen is present.
l…The match lights.

s	o	l
1	1	1
1	0	0
0	1	0
0	0	0

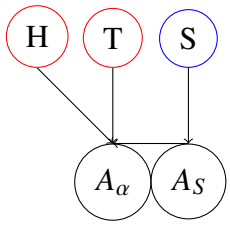
Kaufmann (2013) develops causal premise semantics to translate the information from the causal model to the parameters of the standard Kratzerian framework.[12] The presumed values of the background variables (and, I will assume, general incompatibilities between values) are recorded in the modal base at the world of evaluation. Therefore, for a world w, where the match is not struck but oxygen is present, we obtain $f(w) = \{\neg s, o\}$ from the modal base f. Normal settings of exogeneous variables together with the information in the causal dependency table serve as the ordering source, reflecting what courses of events are considered causally normal. In the case under consideration, we obtain $g(w) = \{o, (s \wedge o) \leftrightarrow l\}$. Thus, $O(f, g, w)$ constitutes the set of worlds compatible with the relevant facts at w that are most normal according to causal dependencies as holding at w. For our specific scenario, we obtain $O(f, g, w) \subseteq \neg l$, meaning that, ignoring subtleties of temporal and aspectual information, *The match will not light* comes out true.

To reflect action choices, we add a set of intervention variables, causally relevant bipartitions conceived of us under the control of the relevant agent. They have no ancestors, and their values are not observed but manipulated (Galhotra and Halpern, 2024). For instance, let us assume

[12]The switch from ordering semantics to premise semantics is inconsequential for our purposes; they identify the same set of optimal worlds.

that agent α's goals is to get from Asakusa to Tawaramachi, which they could achieve either by subway or by walking there. Then, the network can be represented as in the following figure; the intervention variables are H ('Does α walk to Asakusa?') and T ('Does α board the subway to Asakusa?').

s	...	Subway departs for Asakusa
a_S	...	Subway arrives in Asakusa
h	...	Agent walks ('hikes') to Asakusa
t	...	Agent takes subway to Asakusa
a_α	...	Agent arrives in Asakusa



h	t	s	a_S	a_α
0	0	1	1	0
1	0	1	1	1
0	1	1	1	1

I assume a general incompatibility between walking and taking the subway which I take to be recorded by the modal base as '$h \rightarrow \neg t$' and '$t \rightarrow \neg h$', together with the fact that the subway is operating 's'. The table to the right of the graph reflects normalcy assumptions that feed into the ordering source (cases ruled out by the modal base are omitted): the subway arrives in Asakusa if it runs ($s \rightarrow a_S$), if α takes the subway and the subway arrives, so does α ($(t \wedge a_S) \rightarrow a_\alpha$) and if α walks, they arrive ($h \rightarrow a_\alpha$). Finally, we assume that normally, walking and taking the subway are the only strategies the agent can adopt to reach their goal ($(\neg h \wedge \neg t) \rightarrow \neg a_\alpha$). Consequently, in our case, $f(w) = \{h \rightarrow \neg t, t \rightarrow \neg h, s\}$, and $g(w) = \{s, s \rightarrow a_S, (t \wedge a_S) \rightarrow a_\alpha, h \rightarrow a_\alpha\}$. By the strategy of premise semantics, we form maximally consistent sets containing the entire modal base, the designated goal (a_α), and a subset of $g(w)$. The familiar set $O(f, g, w)$ can be derived from these as the union of their intersections, containing worlds where the addressee takes the subway to Asakusa, and worlds where the addressee walks to Asakusa. Assuming that $\gamma(w) = \{h, t\}$ (derived from the intervention variables), the set of optimal worlds is split into two cells depending on the action chosen.

(32) $O^3(f, \gamma, g, w) = \{\bigcap\{a_\alpha, h, \neg t, s, a_S, a_\alpha\}, \bigcap\{a_\alpha, t, \neg h, s, a_S, a_\alpha\}\}$

A teleological possibility statement on its means reading as salient for (33) is correctly predicted to be true in the scenario described:

(33) a. You can take the subway.
b. $\exists W' \in O^3(f, \gamma, g, w)[W' \neq \emptyset \wedge W' \subseteq t]$

5. Local necessity and negation

In the above, we have developed an analysis of possibility modals as expressing necessity with respect to one choosable course of events.[13] At this point, we should return to the status of this local necessity. In this respect, it is particularly interesting to see what happens under negation.[14] Unsurprisingly, the proposed semantics with a non-trivial setting for γ turns out to be rather weak, saying that 'no choosable course of events entails the prejacent'. It is not clear,

[13] As always, the ordering source weakens the statement by shrinking the set of worlds under consideration. This provides a standard Kratzerian solution to Brown's (1988) criticism about a similar proposal to the semantics of ability.

[14] I am indebted to Muyi Yang (p.c.) for pressuring me on this point.

however, if this fits speaker intuitions. Instead, negated ability statements appear to convey negated compatbility, cf. (34).[15]

(34) Mary cannot sing 'First Love' recognizably.
$\Rightarrow$ 'Mary will not sing FL recognizably.'
$\not\approx$ 'It is not the case that she has a strategy to sing FL recognizably.'

To capture speaker intuitions, we could argue that negation involves super-valuation over all partitions into action-choices including the trivial 'nature' one (resulting from γ_N), requiring that on no partition there is a cell that entails the prejacent. Due to the presence of the trival partition, this means that none of the singleton cells entails the prejacent, or equivalently, that the prejacent is not true at any accessible world. Another option to the same effect would be that, in the presence of negation, γ is always set to γ_N. Both assumptions strike me as somewhat stipulative.

Alternatively, we could try to assimilate local necessity to cases of exhaustification. The partitioning we take to underly the non-compatibility readings would then be akin to free choice and distribution (our evidence for plurality), which are known to (usually) vanish under negation.

(35) You can't take Spanish or Calculus.
$\approx$ 'You can take neither Spanish nor Calculus.'
$\not\approx$ 'You don't have a choice between Spanish and Calculus.'

(36) Exactly one girl can take Spanish or Calculus. Aloni (2022), (48)
$\approx$ One girl can choose between the two and each of the others can take neither of them.

Local necessity could, for instance, be derived as local exhaustification of an existential (here, under the plural existential), as proposed for other contexts by Chierchia (2004); Chierchia et al. (2011); Bar-Lev (2021), among others. Before evaluating the feasibility of such a solution, I would like to investigate further cases of negated possibility modals. Interestingly, negated teleological modals are more favorable for the strengthened semantics. Compare (37):

(37) Mary cannot take the A-train.
a. ? $\approx$ If she takes the A-train, she won't reach her goal. ('not compatible')
b. ? $\approx$ There's no way to obtain Mary's goal that would entail that she takes the A-train. ('not a means')

An initial inclination to interpret (37) as negated compatibility (37a) seems easy enough to overwrite. Consider (38).

(38) Mary cannot take the A-train, it doesn't go to Hoboken. If she gets on the A, she will have to switch trains at the next stop ($\not\Rightarrow$ She cannot reach her goal (anymore).)

[15] It is not entirely clear to me that the stronger reading is completely unavailable under negation, consider:
(i) Mary cannot sing First Love recognizably, but she might get lucky and it could come out right for once.
The discussion in the literature suggests strongly that this should be infelicitous (e.g., Thomason, 2003; Nadathur, 2023).

The follow-up suggests that Mary's taking the A-train is not incompatible with her goal (in contrast to what the standard semantics would predict), but just that it is not a means to get to Hoboken (in line with what the plural existential semantics predicts). It is not entailed by any of the strategies, instead, an additional step of switching trains will be required.

The intuitions in favor of the stronger semantics are even clearer in the case of opportunities in situations of practical deliberation, where choosable actions are highly salient by definition. To see this, we can again consider the miners puzzle; (39) repeats the relevant action choices and their possible consequences from (12) above:

(39)

Action	If miners in A	If miners in B
Block shaft A	All saved	All drowned
Block shaft B	All drowned	All saved
Block neither shaft	One drowned	One drowned

Previously, we wanted to predict the truth of (40a) and the falsity of (40b), which is achieved by the semantics as proposed here. To this we now add (40c):

(40) a. We can block shaft A, and we can block shaft B.
b. We can block the shaft they are in.
c. We cannot block the shaft they are in.

As negated compatibility, (40c) would be false; the miners are in A or in B, and for either case there are accessible worlds where we block that one. Yet, speakers consider (40c) true. And in fact, no cell corresponding to an action taken (block A or block B) entails that we block the shaft they are in. The interpretation of *can* as a plural indefinite with local necessity makes the correct prediction in this case.

While opportunity modals and teleological modals provide evidence that the proposed semantics is correct also in interaction with negation, negated ability modals remain problematic for reasons that are still unclear to me.

Considerations along these lines strike me as particularly relevant for comparing my proposal to Nadathur's (2023) proposal, which is developed for ability modals specifically and relies crucially on causal sufficiency and necessity. She uses a combination of presuppositions and at-issue meaning to explain the implicative behavior in the positive and negative case. Ability is intended to describe a hypothetical guarantee (for related approaches, see Louie, 2014; Mandelkern et al., 2017). Her proposal is sketched in (41b) (her (7.10)): the at-issue content (compatibility) is coupled with the presupposition that there be a predicate A that is causally necessary and sufficient for the prejacent:

(41) x Able P
a. presupposes the existence of a predicate A s.t.
(i) $A(x)$ is causally necessary for $P(x)$, (ii) and $A(x)$ is causal sufficient for $P(x)$
b. at-issue content: $\Diamond A(x)$

Thanks to the presupposition, at any world where A is carried out by x, the prejacent will come

true, and carrying out A is possible for the agent. Negation targets the circumstantial possibility of the causally necessary $A(x)$. Despite the set-up in a causal model, the at-issue semantics does not go beyond the standard treatment in terms of compatibility. While thus unproblematic in the negative case, it is too weak in the positive case, and any attempt to strengthen the interpretation of $\diamond$ in the at-issue semantics risks issues with negation that I observed for the account I proposed for possibility modals in general.

6. Conclusions

I have proposed a unified semantics for priority and dynamic possibility modals as plural existential quantifiers with local necessity. I have assumed that the domain of the existential quantification depends on choices taken by a relevant agent. This parameter can, however, be trivialized to singleton sets of worlds when action choices are not relevant ('nature' chooses). The universal quantifier in the scope of the plural existential quantifier (local necessity) can be understood as reflecting distribution of a singular predicate of worlds (the prejacent) to the plural witness of the existential quantification. I showed that this yields a potentially too weak semantics under negation, but concluded that enough cases warrant the stronger semantics even under negation. As in the positive case, the cases where mere compatibility is desirable can always be explained in terms of a trivialization of the action choices to the set of all propositions. For future work it would be desirable to compare in more detail with solutions devised specifically for ability modals (e.g. Nadathur, 2023; or Mandelkern et al., 2017, with a refined ontology allowing for quantification over acts). Different ways of layering action choices and ordering are worth exploring, as well as the comparison with X-marked possibility modals like *could* (see von Fintel and Iatridou, 2023; Ferreira, 2023). Finally, it would be interesting to hold the solution proposed here for dynamic and priority modals against compatbility issues in the epistemic domain. The clustering by action choices seems intuitively similar to coarsening of the logical space as modeling awareness (Franke and de Jager, 2007; Li, 2009). I will leave it for future research if this might allow further unification across differently flavored possibility modals.

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